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A R A B L E A G U E

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A Simplified Procedure For Calculating Torsional Stresses In Container Ships

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SUMMARY:

A simplified procedure is given for calculating the shear and warping stresses as well as the torsional deformations induced in container ships by torsional loading. Particular attention being paid to the maximum values induced at the aft and forward ends of the open length of a ship. The procedure is based on the sectorial properties of thin-walled uniform members.

The ship section is idealized by a simplified configuration so as to calculate the sectorial properties, shear centre and warping constant.

The solution of the torsion equation is given for the general case of a polynomial torque distribution as well as for the particular cases of linear and parabolic torque distributions. The solution takes into account the degree of fixity provided by the closed structures at both ends of the ship. The particular solution for open ships having low torsional rigidity is examined. A numerical example based on a linear torque distribution is given so as to illustrate the calculation procedure.

It is concluded that the development of a simplified procedure for the structural analysis and design of open ships could be very useful in the preliminary design stages as well as for comparing alternative designs, particularly when the torsional loading cannot be accurately determined. The use of the finite element method should be confined only to the final stages of design.

INTRODUCTION:

The demand on container ships is increasing very rapidly as a result of the expansion in world trade and the general increase in the application of containerization systems [1, 2] .

Container ships are characterised by exceptionally wide hatch openings, which may exceed 80% of ship breadth. These wide hatches have a significant effect on the torsional strength and rigidity of ship hull girder [3] .Torsional loading, therefore, induces additional stresses, normally called warping stresses, near hatch corners [4].

Torsional loading may result from the motion of a ship in oblique waves. Improper cargo loading also induces torsional loading. Open ships are subjected to additional torsional loading induced by the horizontal component of the shearing force. The latter acts at a distance from the shear centre of the ship section.

The torsional behaviour of open ships could be studied theoretically, using the finite element method [3,5] , or experimentally, using model testing [6-8] . A simplified approach, based on the torsional behaviour of open sections, could be also used [9,10] . However, correlation with full-scale measurements [11] becomes necessary in order to confirm the validity of the method of analysis.

Since the finite element method and model testing are costly and time consuming, their use should be confined to the final analysis of the structure.

However, in the preliminary design stages, for comparing different designs and when the torsional loading cannot be accurately determined, a simplified procedure for the structural design and analysis of open ships is very desirable.

In this paper, a general procedure is given for calculating the shear and flexural warping stresses as well as the torsional deformations induced in open ships by torsional loading. The method is based on the sectorial properties of the ship section, which is idealized by a simplified configuration.

The solution of the torsion equation is given for the general case of a polynomial torque distribution as well as for the particular cases of linear and parabolic torque distributions. The solution takes into account the degree of fixity provided by the closed structures at both ends of the open length of the ship. Open ships having relatively low torsional rigidity is especially considered.

A numerical example, based on a linear torque distribution, is given so as to illustrate the calculation procedure.

1- Calculation of the shear and flexural warping stresses.

The shear and flexural warping stresses are calculated according to reference [12] as follows:

$$\tau(\omega) = \frac{T(\omega) . S(\omega)}{J(\omega) . t} \qquad (1)$$

$$\sigma(\omega) = \frac{M(\omega).\omega}{J(\omega)} \qquad (2)$$

where, $M(\omega)$ = Bimoment

$$= - E.J(\omega).\frac{d^2\varphi}{dx^2} \qquad (a)$$

$T(\omega)$ = warping torsional moment.

$$= - E.J(\omega).\frac{d^3\varphi}{dx^3} \qquad (b)$$

$J(\omega)$ = warping constant of ship section.
= sectorial moment of inertia [13]

ω = sectorial coordinate.
E = modulus of elasticity.
φ = angle of twist.
t = thickness.

$S(\omega)$ = sectorial static moment.

Therefore, in order to calculate the warping stresses, the following information is required:

i - the sectorial properties of the ship section, i.e.

ω, $S(\omega)$ and $J(\omega)$.

ii - the solution of the torsion equation so as to calculate $M(\omega)$ and $T(\omega)$.

2 - Solution of the torsion equation.

The differential equation of the combined bending and torsion [14] is given by:

$$E.J(\omega).\ \frac{d^3\varphi}{dx^3} - G.J_t.\ \frac{d\varphi}{dx} = -T_x \qquad (3)$$

where,

T_x = torsional moment.

x = length coordinate.

$G.J_t$ = torsional rigidity.

J_t = Saint-Venant torsional constant.

$E.J(\omega)$ = warping rigidity.

The solution of equation (3) depends on the torque distribution, T_x , and the boundary conditions.

i - Boundary conditions.

There are three general cases of boundary conditions:

a) fixed end i.e. no warping.

This is satisfied by:

$$\varphi = \frac{d\varphi}{dx} = 0$$

b) Free end, i.e. free warping.

This is satisfied by:

$$\frac{d^2\varphi}{dx^2} = 0$$

c) Constrained end, i.e. constrained warping.

$$\frac{d\varphi}{dx} = \frac{T_e}{G.J_t} \cdot (1-f)$$

where T_e = torsional moment at the end of the member, and

$$0 \leqslant f \leqslant 1.0$$

The magnitude of "f", for any particular ship, depends on the configuration of the end structure, and could be determined either experimentally or using reference [7] .

ii - Distribution of torsional loading.
The magnitude and distribution of the torsional loading acting on a ship hull girder depend on several factors, among them are the sea condition, shape of ship form and the location of the shear centre. Different approaches, theoretical, experimental and empirical, have been proposed for the estimation of the torsional loading.
However, there is obvious disagreement among various authors, as illustrated by Meek [11]. Consequently, in the following analysis, a simplified torque distribution, based on a polynomial representation, is used, i.e.:

$$T_x = \sum_{i=0}^{n} b_i \cdot x^i$$

The general solution of the torsion equation could be obtained by summing the solution of the various terms of the polynomial, as given in Appendix (1).

It should be indicated here that any torque distribution could be represented by a polynomial equation, using curve fitting techniques. Therefore, the assumption of polynomial representation is valid, irrespective of the shape of the torque distribution.

However, in order to simplify the analysis, two simple torque distributions are considered:

a) Linear torque distribution.

$$T_x = T_o \cdot \left[1-(1-\alpha)\cdot\frac{x}{a}\right] \qquad \text{(c)}$$

b) Parabolic torque distribution

$$T_x = T_o \cdot \left[1-(1-\alpha)\cdot\left(\frac{x}{a}\right)^2\right] \qquad \text{(d)}$$

In both cases, the origin of x is at mid-length, see Fig. (1), and T_o is the maximum value of the torque at x = 0.

The magnitude of T_o could be obtained either from Numata [15] :

$$T_o = 0.01 \cdot \mu \cdot L \cdot B^3 \quad \text{t.m.}$$

or according to De Wilde [7] :

$$T_o = C_t(1.75 - 1.5 . \frac{e_Y}{D}).L.B^3 . e^{-\frac{L}{1900}} \text{ t.f.}$$

where, μ = a coefficient which varies from 0.24-0.34,

e_Y = distance of shear centre from idealized bottom,

L = ship length,

D = ship depth,

C_t = a coefficient depending on waterplane area coefficient, C_w , as follows [16] :

C_w	0.6	0.7	0.8	0.9
$C_t \times 10^{-4}$	0.384	0.531	0.716	0.942

iii- Solution of the torsion equation.
Assuming constrained warping at both ends of the open length, see Fig. (1), the boundary conditions are:

$$\text{at } x = 0 , \varphi = \frac{d^2\varphi}{dx^2} = 0, \text{ and}$$

$$\text{at } x = a , \frac{d\varphi}{dx} = \frac{T_e}{G.J_t} (1 - f)$$

Using these boundary conditions, the solution of equation (3) is obtained for the linear and parabolic torque distributions as follows:

a) Linear torque distribution.
The solution of the torsion equation is given by:

$$\varphi = \frac{T_o \cdot a}{G \cdot J_t}\left[\frac{x}{a} - \tfrac{1}{2}(1-\alpha)\left(\frac{x}{a}\right)^2 - f \cdot \alpha \cdot \frac{\sinh kx}{ka \cdot \cosh ka} - \frac{(1-\alpha)}{(ka)^2} \cdot \eta\right] \qquad (4)$$

$$M(\omega) = T_o \cdot a \cdot \left[f \cdot \alpha \cdot \frac{\sinh kx}{ka \cdot \cosh ka} + \frac{(1-\alpha)}{(ka)^2} \cdot \eta\right] \qquad (5)$$

$$T(\omega) = T_o \cdot \left[f \cdot \alpha \cdot \frac{\cosh kx}{\cosh ka} + \frac{(1-\alpha)}{ka}\left\{\tanh ka \cdot \cosh kx - \sinh kx\right\}\right] \qquad (6)$$

where, $k^2 = \dfrac{G \cdot J_t}{E \cdot J(\omega)}$

$$\eta = 1 + \tanh ka \cdot \sinh kx - \cosh kx$$

b) Parabolic distribution.
The solution of equation (3) gives:

$$\varphi = \frac{T_o . a}{G . J_t}\left[\frac{x}{a} - \frac{(1-\alpha)}{3}\left(\frac{x}{a}\right)^3 - f . \alpha . \frac{\sinh kx}{ka . \cosh ka} - \frac{2(1-\alpha)}{(ka)^2}\left\{\frac{x}{a} - \frac{\sinh kx}{ka . \cosh ka}\right\}\right] \qquad (7)$$

$$M(\omega) = T_o . a\left[f . \alpha . \frac{\sinh kx}{ka . \cosh ka} + \frac{2(1-\alpha)}{(ka)^2}\left\{\frac{x}{a} - \frac{\sinh kx}{ka . \cosh ka}\right\}\right] \qquad (8)$$

$$T(\omega) = T_o\left[f . \alpha . \frac{\cosh kx}{\cosh ka} + \frac{2(1-\alpha)}{(ka)^2}\left\{1 - \frac{\cosh kx}{\cosh ka}\right\}\right] \qquad \ldots\ldots(9)$$

The maximum values of $M(\omega)$ and $T(\omega)$ are given in Appendix (2). When J_t is relatively small in comparison with $J(\omega)$, the solution of equation(3) could be simplified as shown in Appendix(3).

3- Calculation of the sectorial properties.
The sectorial properties of a section of an open ship could be calculated using the general procedure given in reference [13] , as follows:

a) Idealization of a ship section.
A section of the double-skin structure of an open ship, see Fig. (2), could be idealized by a simplified open section, as shown in Fig.(3). The dimensions of the idealized section is obtained from the original configuration as indicated in reference [7] .

b) Position of shear centre:

Due to symmetry about the Y-axis, the shear centre is located on the Y-axis at a distance e_Y from P, see Fig.(3). The distance e_Y is given by [12] :

$$e_Y = \frac{1}{I_Y} \int_A Z \cdot \omega \cdot dA \qquad (10)$$

Where, $$I_Y = 2A_w \cdot \left(\frac{h}{2}\right)^2 + A_b \cdot \frac{b^2}{12} + 2 \cdot \sum_{i=1}^{n} A_i \cdot Z_i^2 + 2 \cdot \sum_{j=1}^{m} A_j \cdot \left(Z_j^2 + \frac{p_j^2}{12}\right) \qquad (11)$$

2b = B - -

t_w; t_b = Effective thickness of the side and bottom respectively.

A_i = sectional area of girder i.

A_b; A_w - sectional area of the bottom and side respectively.

2m, 2n = number of girders in the side and bottom respectively.

ω sectorial coordinate based on an assumed pole at P, see Fig.(4).

From Fig.(4) we have:

$$\int_A Z.\omega'.dA = b^2.d^2.t_w/4 + b.\sum_{i=1}^{m} \omega'_i.A_i \qquad (12)$$

$$\text{Hence, } e_Y = (b^2.d^2.t_w/4 + b.\sum_{i=1}^{m} \omega_i.A_i)/I_Y \qquad (13)$$

c) Principal sectorial area diagram.

Having determined the location of the shear centre, the principal sectorial area diagram could be determined as follows:

$$\omega = -\int_0^{b/2} e_Y.ds, \quad \text{for the bottom}$$

$$\omega = -e_Y.b/2 + \int_0^{d} \frac{b}{2}.ds, \text{ for the side}$$

These calculations are represented graphically in Fig.(5).

d) Warping constant J(ω).

The warping constant J(ω) is calculated as follows [13]:

$$J(\omega) = \int_A \omega^2.dA$$

$$= 2.\sum_{i=1}^{m+n} \omega_i^2.A_i + \frac{A_b.b^2}{12}.e_Y^2$$

$$+ \frac{b^2 \cdot t_w}{6} \left[e_Y^3 - (e_Y - d)^3 \right] \qquad (14)$$

e) <u>Sectorial static moment, $S(\omega)$.</u>

The sectorial static moment is calculated as follows:

$$S(\omega) = \int_A \omega \, dA + \sum_A \omega_i \cdot A_i, \quad i=1,2,\ldots,(m+n)$$

For the side, $S(\omega)$ is given by:

$$S(\omega) = \frac{b \cdot t_w}{2} \left\{ \xi_1 (d - e_Y) - \frac{\xi_1^2}{2} \right\}$$

where, $0 \leqslant \xi_1 \leqslant d$, see Fig.(6).

For the bottom, $S(\omega)$ is given by:

$$S(\omega) = \frac{b \cdot d \cdot t_w}{4} (d - 2 \cdot e_Y) - \left(\xi_2 - \frac{\xi_2^2}{b}\right) \cdot \frac{b \cdot t_b}{2} \cdot e_Y$$

where, $0 \leqslant \xi_2 \leqslant b/2$

The distribution of $S(\omega)$ over half [redacted] ship section is shown in Fig.(6).

f) <u>Torsional constant, J_t</u>

The torsional constant for a double skin structure is calculated from the original ship section as follows:

$$J_t = \sum_{i=1}^{r} 4.A_i^2 \Big/ \oint \frac{ds}{t}$$

where r = number of cells,

A_i = enclosed area of cell i.

4 -Numerical example.

Consider an open ship having the following main dimensions,

L = 152.9 m	B = 26.0 m
D = 16.2 m	h = 1.6 m
p = 2.2 m	2a=120.0 m, see Fig.(2).

The idealized section has the following dimensions:

d = 15.4 m	b = 23.8 m
t_w = 30 mm	t_b = 37 mm, See Fig.(3).

I- <u>Results</u>:

The results of the various calculations, for the case of the linear torque distribution, are given in Tables(1,2)and are shown in Figs.(7,8). These results are based on Numata's value of T_o and on the following assumptions:

$\mu = 0.3 \qquad C_t=0.7 \qquad \alpha=0.25 \qquad f=0.8$

The Saint-Venant shear stress is given by;

$$\tau_s = 0.227 \qquad t/cm^2$$

Thus the maximum total shear stress resulting from torsion loading is given by:

$$\tau_t = 0.36 \qquad t/cm^2$$

II-Discussion:

Form the results of calculations, it is shown that:

a) The maximum values of $\sigma(\omega)$ occur in the deck plating, where σ_H, the normal stress induced by the bending of the hull girder, may attain high values.

b) The total normal stress, $\{\sigma_H + \sigma(\omega)\}$, may be further increased, near hatch corners, by virtue of stress concentration [9].

c) The maximum total shear stress, resulting from torsional loading, occurs in the same region where the shear stress, induced by the longitudinal vertical shearing force, may attain high values [17]. The maximum shear warping stress is of the order of 5% of the Saint-venant shear stress.

d) Open ships having insufficient torsional rigidity may experience large angle of twist at both ends of the open length.

e) The degree of constraint provided by the closed structures at the end of the open length has a marked influence on the magnitudes of the flexural warping stress and the bending stresses developed in the transverse deck strips between hatches. A reduction of 17% in the flexural warping stress results from a 50% reduction in the degree of fixity provided by the end structure.

f) Apart from the increased shear and warping stresses, very wide hatch openings may also have an adverse effect on the transverse strength of container ships.

5 - <u>Concluding remarks:</u>

The main conclusions drawn up from this investigation could be summarised as follows:

i) Although the direct approach for the stress analysis of ship structures is the finite element method, the time and cost required for data generation, computations and analysis of results makes it very desirable to have an alternative simplified approach for preliminary design and for comparing alternative designs.

This is particularly important when the external loading cannot be accurately determined, as in the case of open ships.

ii- Assuming the open length of the hull girder to be a uniform member, the shear and warping stresses, induced by torsional loading, could be calculated using the sectorial properties of the ship section. The magnitude of these stresses depends on the magnitude and distribution of the torsional loading, position of the shear centre and torsional and warping rigidities of the ship section.

iii- The constraining effect provided by the closed structures at both ends of the ship has a marked influence on the magnitude of the flexural warping stresses and the bending stresses developed in the transverse deck strips between hatches.

iv- Double skin structures provide greater torsional strength than single-skin structures.

v- Container ships may experience high shear and bending stresses in the side and deck plating.

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APPENDIX (1)

Solution of the general torsion equation

The general torsion equation is given by:

$$\varphi''' - k^2 . \varphi' = - T_x/C_1 \qquad A(1.1)$$

Where, T_x = External torque,

C_1 = Warping rigidity,

$k = \sqrt{C/C_1}$,

C = Torsional rigidity.

The solution is given in the form:

$$\varphi(x) = \varphi_1(x) + \varphi_2(x) \qquad A(1.2)$$

where, $\varphi_1(x)$=Complementary function, and is given by:

$$\varphi_1(x) = A_o + A_1 .cosh\ kx + A_2 .sinh\ kx \qquad A(1.3)$$

where, A_i, (i=0,1,2), are constants to be determined from the support boundary conditions.

$\varphi_2(x)$ =particular integral, P.I., which depends on the function of the torque T_x.

If T_x is given in a polynomial form; i.e.

$$T_x = \sum_{i=0}^{n} b_i .x^i \qquad A(1.4)$$

The P.I. could be obtained for any individual term of the polynomial. The required $\phi_2(x)$ is obtained by summing up the P.I. of each term in the polynomial.

Assuming the general term of the torque equation to be:

$$T_r = b_r \cdot x^r \qquad A(1.5)$$

and substituting in equation A(1.1) we get:

$$(D^3 - k^2 . D) . \phi = - \frac{b_r}{C_1} . x^r$$

Hence, $$\phi_2(x) = \frac{D^{-1}}{k^2} \left[1 - \left(\frac{D}{k}\right)^2 \right]^{-1} . B_r . x^r$$

$$= \left[\frac{D^{-1}}{k^2} + \frac{D}{k^4} + \frac{D^3}{k^6} + \frac{D^5}{k^8} + \dots \right] . B_r . x^r$$

$$= B_r . \left[\frac{x^{r+1}}{k^2(r+1)} + \frac{r . x^{r-1}}{k^4} + \frac{r(r-1)(r-2) . x^{r-3}}{k^6} + \frac{r(r-1)(r-2)(r-3)(r-4) . x^{r-5}}{k^8} + \dots \right] \qquad \dots\dots\dots A(1.6)$$

where, $B_r = b_r / C_1$

The P.I. for (r = 0, 1, ..., 5) is given in table(3), from which $\phi_2(x)$ could be obtained by summing the appropriate terms of the polynomial. However, if the torque distribution is given in any other form, a polynomial equation could be fitted and the P.I. could be obtained form Table (3).

APPENDIX (2)

Maximum values of $M(\omega)$, $T(\omega)$ and φ

1.-Linear torque distribution.

a) Angle of twist.

The maximum value of φ occurs at x=a. Substituting in equation (4) of the text we get:

$$\varphi_a = \frac{T_o . a}{G . J_t} \left[(\tfrac{1}{2} - \beta) + \alpha (\tfrac{1}{2} - \frac{f}{ka} . \tanh ka + \beta) \right]$$

A(2.1)

where, $\beta = \frac{1}{(ka)^2} (1 + \tanh ka . \sinh ka - \cosh ka)$

b) Bimoment.

The maximum value of $M(\omega)$ occurs at x=a. Substituting in equation (5) of the text we get:

$$M(\omega)_a = T_o . a \left[\frac{f . \alpha}{ka} . \tanh \; ka + \frac{(1-\alpha)}{(ka)^2} . \beta \right]$$

....A(2.2)

c) Warping torsional moment, $T(\omega)$.

The maximum value of $T(\omega)$ occurs either at x=0 or at x=a depending on the value of α. Substituting in equation (6) of the text we get:

$$T(\omega)_o = T_o\left[\frac{f\cdot\alpha}{\cosh ka} + \frac{(1-\alpha)}{ka}\cdot\tanh ka\right] \qquad A(2.3)$$

and $$T(\omega)_a = T_o\cdot f\cdot\alpha \qquad A(2.4)$$

For small values of , $T(\omega)_o > T(\omega)_a$

2- Parabolic torque distribution.

a) Angle of twist.

At $x = a$, equation (7) gives:

$$\varphi_a = \frac{T_o\cdot a}{G\cdot J_t}\left[\left(\frac{2}{3} - \delta\right) + \alpha\left(\frac{1}{3} - \frac{f}{ka}\cdot\tanh ka + \delta\right)\right] \quad \ldots A(2.5)$$

Where, $\delta = \frac{2}{(ka)^2}\left(1 - \frac{\tanh ka}{ka}\right)$

b) Bimoment.

Substituting $x=a$ in equation (8) we get:

$$M(\omega)_a = T_o\cdot a\left[\frac{f\cdot\alpha}{ka}\cdot\tanh ka + \frac{2(1-\alpha)}{(ka)^2}\left(1 - \frac{\tanh ka}{ka}\right)\right] \quad \ldots A(2.6)$$

c)Warping torsional moment, $T(\omega)$

Substituting $x=0$ in equation (9), we get:

$$T(\omega)_o = T_o \left[\frac{f.\alpha}{\cosh ka} + \frac{2(1-\alpha)}{(ka)^2} \left(1 - \frac{1}{\cosh ka} \right) \right] \qquad A(2.7)$$

and for x=a, we get:

$$T(\omega)_a = T_o . f . \alpha \qquad A(2.8)$$

APPENDIX (3)

Solution of the torsion equation when J_t is very small.

When J_t is very small, compared with $J(\omega)$, equation (3) in the text becomes:

$$E.J(\omega). \frac{d^3\varphi}{dx^3} = - T_x \qquad A(3.1)$$

Assuming that the member is fixed at x=a, the solution of equation A(3.1), for both cases of torques distribution, is as follows:

a) Linear torque distribution.

$$\varphi = - \frac{T_o}{E.J(\omega)} \left[\frac{x^3}{6} - (1-\alpha). \frac{x^4}{24a} + \left(\frac{a^2}{3} + \frac{\alpha.a^2}{6} \right).x \right] \qquad A(3.2)$$

At x=a, we have:

$$\varphi_a = \frac{T_o \cdot a^3}{24\, E \cdot J(\omega)} \cdot (5 + 3\alpha) \qquad A(3.3)$$

and $$M(\omega)_a = T_o \cdot a\,(1 + \alpha)/2 \qquad A(3.4)$$

b - Parabolic torque distribution.

$$\varphi = - \frac{T_o}{E \cdot J(\omega)} \left[\frac{x^3}{6} - (1-\alpha) \cdot \frac{x^5}{60\, a^2} + \left(\frac{5}{12} + \frac{\alpha}{12} \right) \cdot a^2 \cdot x \right] \qquad A(3.5)$$

At $x = a$, we have:

$$\varphi_a = \frac{T_o \cdot a^3}{15 E \cdot J(\omega)} \cdot (4 + \alpha) \qquad A(3.6)$$

$$M(\omega)_a = T_o \cdot a\,(2 + \alpha)/3 \qquad A(3.7)$$

Table (1)

Sectorial properties

i	1	2	3	4	5	6
A_i m^2	0.02165	0.02[illegible]	0.022	0.022	0.0264	[illegible]
Z_i m	[illegible].25	4.05	1.9	11.[illegible]	.9	.9
ω_i' m^2	——	——	46.1	93.0	154.6	133.[illegible]
ω_i m^2	-6.59	-40.7	-34.0	12.7	74.8	103.2
$\omega_i.A_i$ m^4	-0.186	-0.882	-0.748	0.281	1.97	5.44
$\int \omega.dA$ m^4	0.0	6.405	13.1	12.0	- 7.77	-12.1

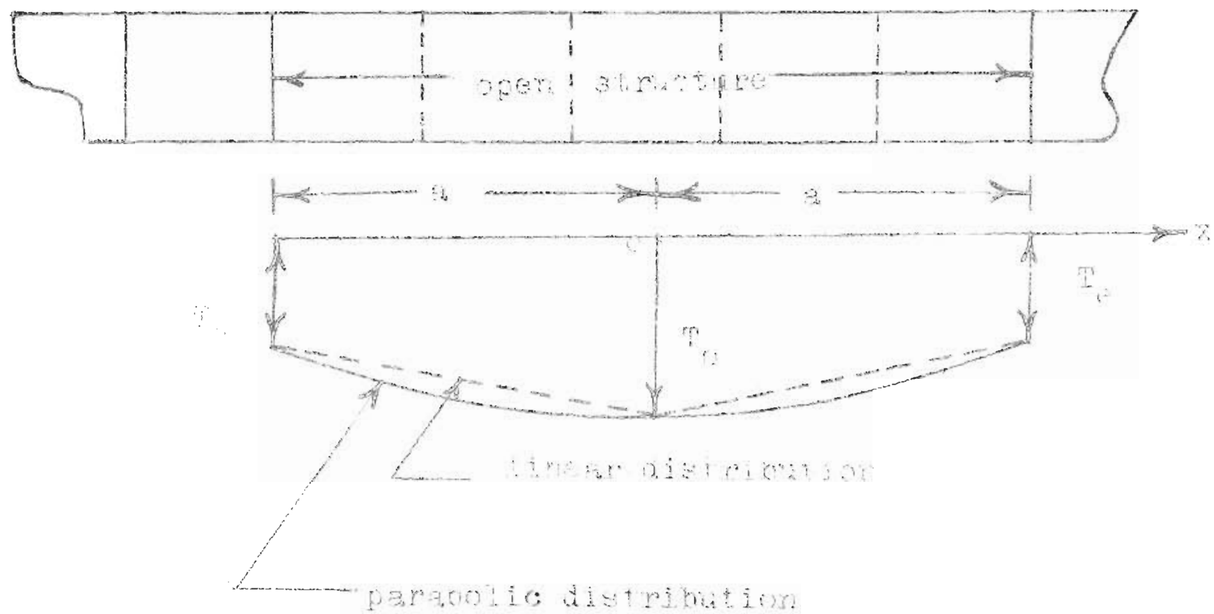
Table (2)
Results of calculations

Item	Numerical value
I_Y	208.91 m^4
[illegible]$_Y$	6.73 m
$J(\omega)$	6150.0 m^6
J_p	6.41 m^4
k	0.02 m^{-1}
T_o (Numata)	8100.0 t.m.
T_o (De Wilde)	5100.0 t.m.
$M(\omega)_{max}$	1.94×10^5 t.m.^{2}
$T(\omega)_{max}$	5050.0 t.m.
$\sigma(\omega)_{max}$	0.40 t/cm^2
$\tau(\omega)_{max}$	0.[illegible]33 t/cm^2
φ_{max}	$0.2[illegible]4 \times 10^{-2}$ radians

Table (3)

Particular Integral for $T_r = b_r . x^r$

r	T_r	$_2(x)$
0	b_o	$\frac{b_o}{C} . x$
1	$b_1 x$	$\frac{b_1}{C}(\frac{x^2}{2} + \frac{1}{k^2})$
2	$b_2 x^2$	$\frac{b_2}{C}(\frac{x^3}{3} + \frac{2x}{k^2})$
3	$b_3 x^3$	$\frac{b_3}{C}(\frac{x^4}{4} + \frac{3x^2}{k^2} + \frac{6}{k^4})$
4	$b_4 x^4$	$\frac{b_4}{C}(\frac{x^5}{5} + \frac{4x^3}{k^2} + \frac{24x}{k^4})$
5	$b_5 x^5$	$\frac{b_5}{C}(\frac{x^6}{6} + \frac{5x^4}{k^2} + \frac{60x^2}{k^4} + \frac{120}{k^6})$



$T_e = \alpha T_0$

Fig.(1)
ASSUMED TORQUE DISTRIBUTION

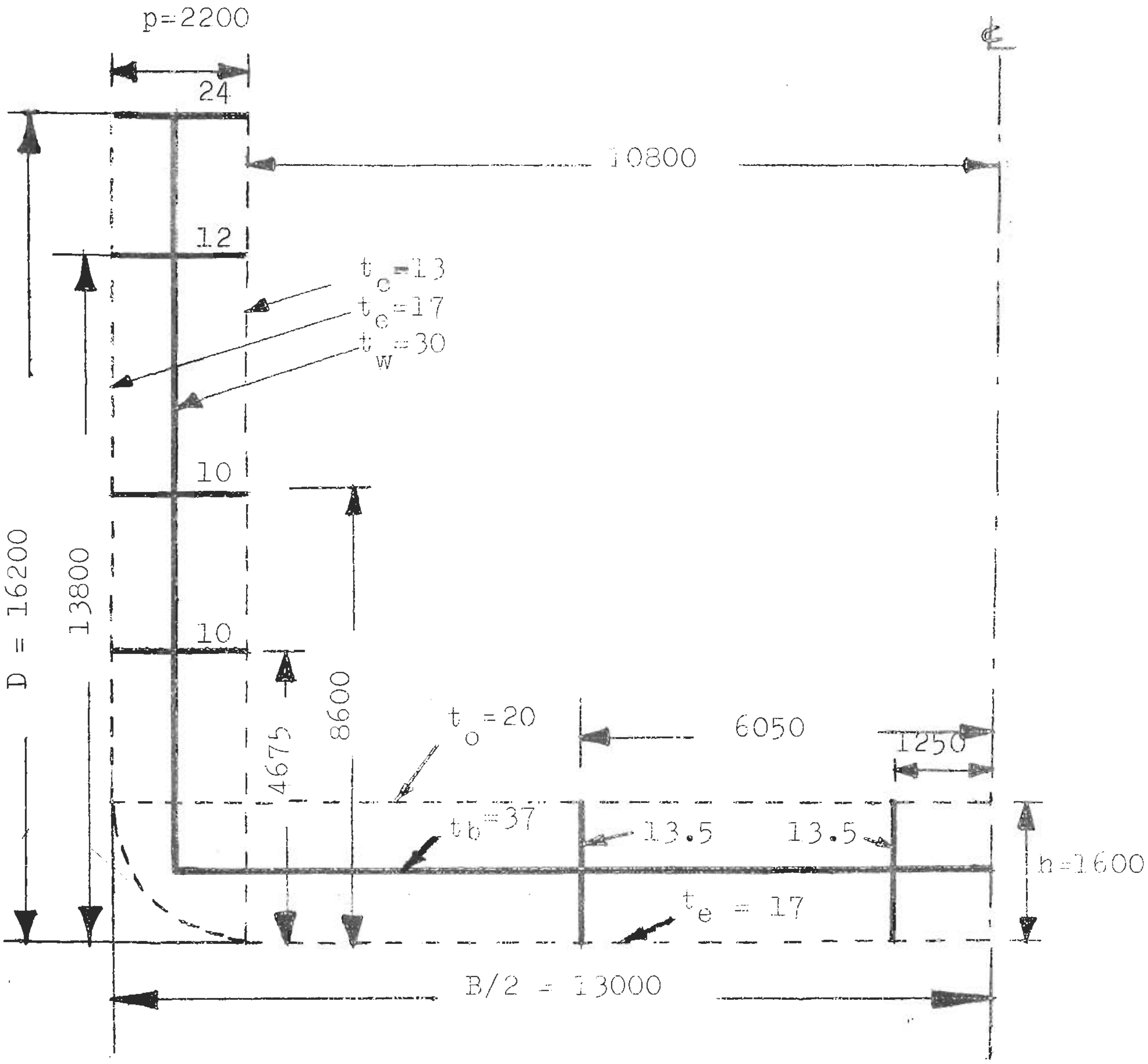


Fig.(2). A SECTION OF AN OPEN SHIP

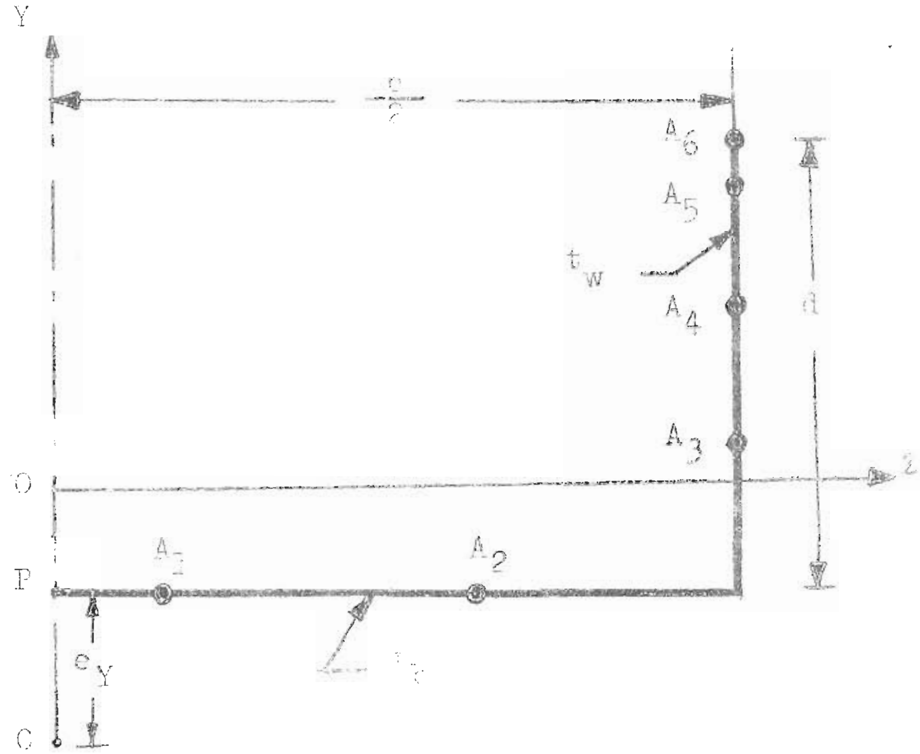


Fig.(3). IDEALIZED SHIP SECTION

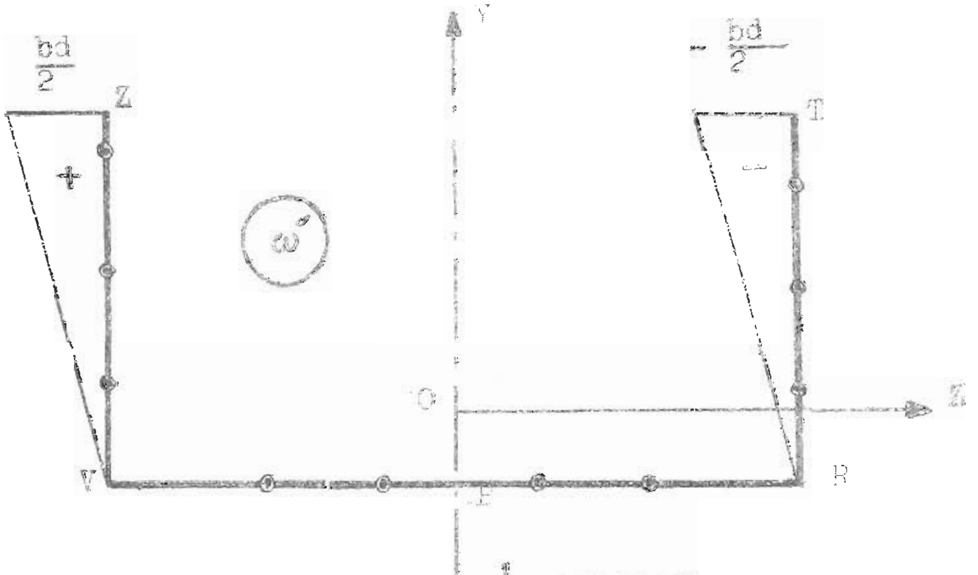


Fig.(4). ASSUMED ω' DIAGRAM

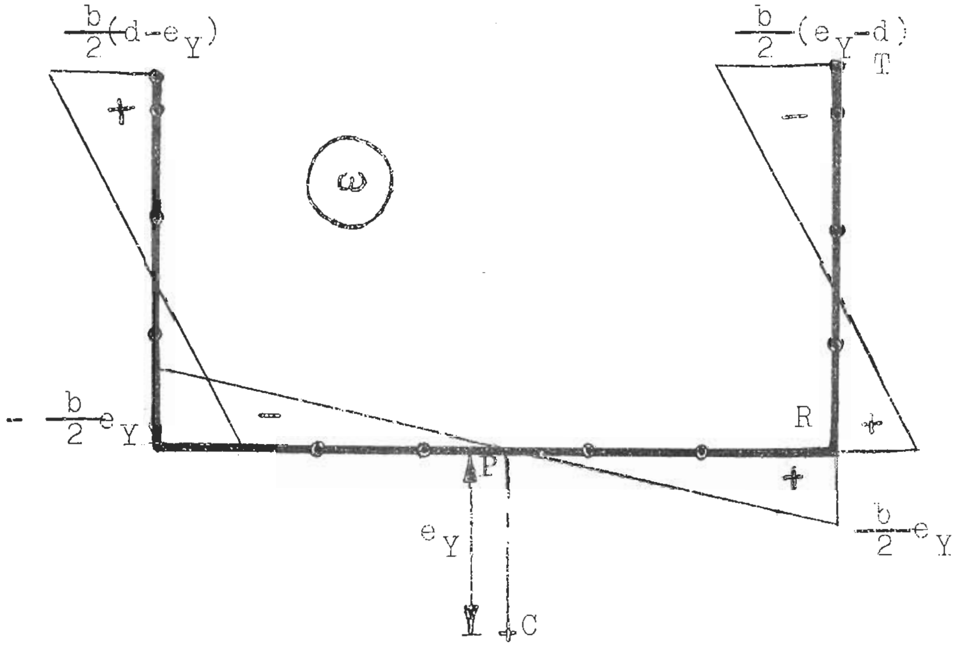
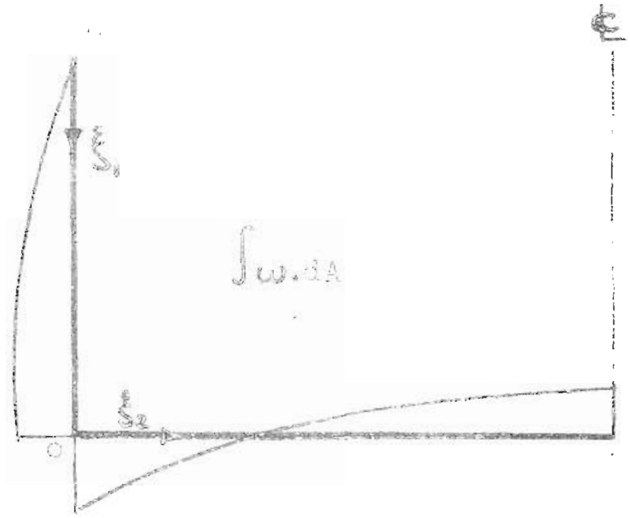


Fig.(5). PRINCIPAL SECTORIAL
AREA DIAGRAM



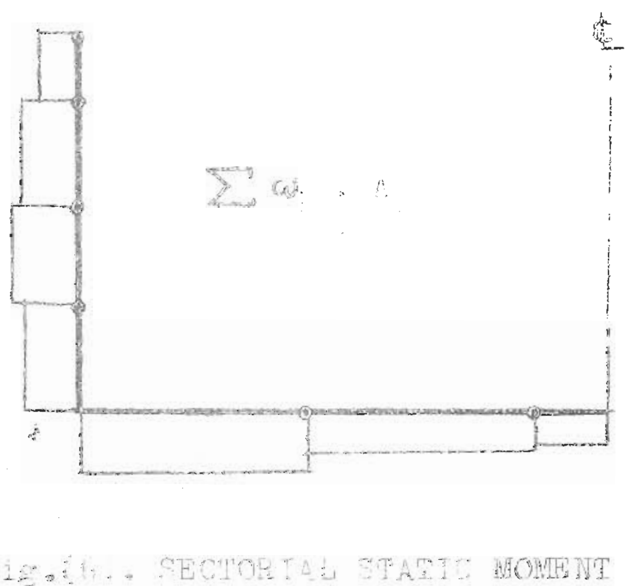


Fig.16.. SECTORIAL STATIC MOMENT

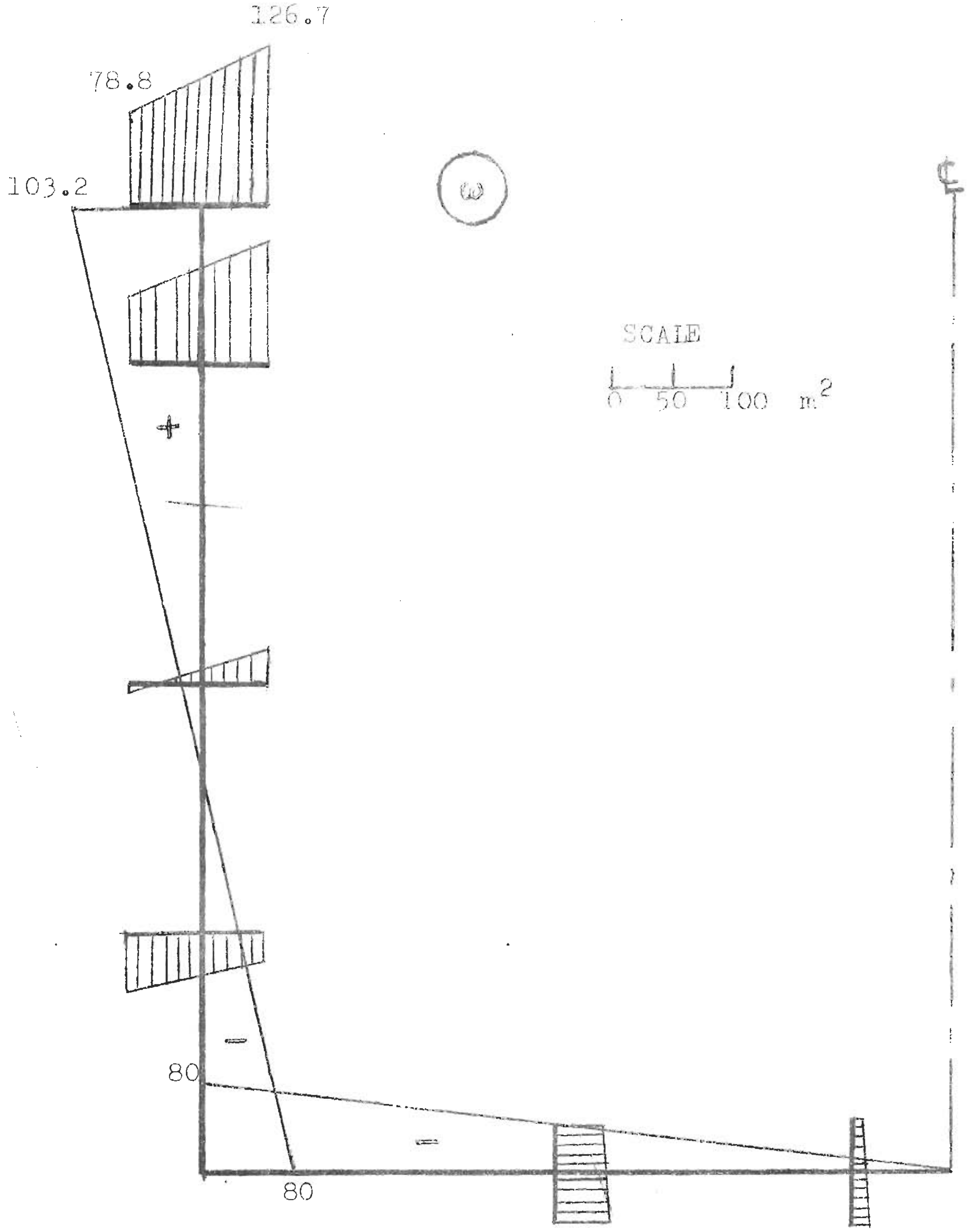


Fig.(7). PRINCIPAL SECTORIAL AREA DIAGRAM

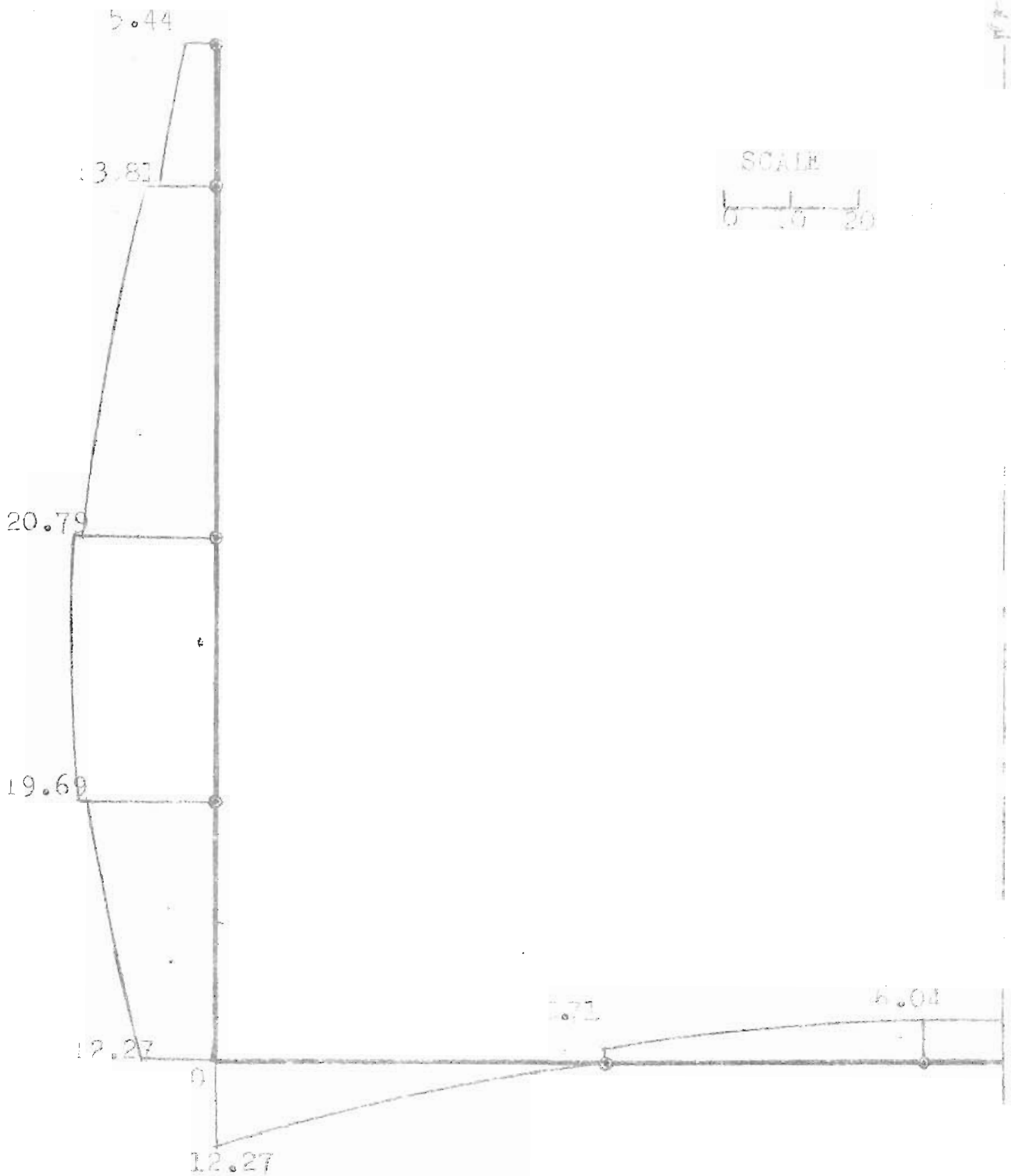


Fig.(11). SECTORIAL STATIC MOMENT